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Gravitational Effects of Global Textures

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ABSTRACT

A solution for the dynamics of global textures is obtained. Their gravitational field during the collapse and the subsequent evolution is found to be given solely by a space-time dependent "deficit solid angle". The frequency shift of photons traversing this gravitational field is calculated. The space-time dependent texture metric locally contracts the volume of three-space and thereby induces overdensities in homogeneous matter distributions. There are no gravitational forces unless matter has a nonzero angular momentum with respect to the texture origin which would be the case for moving textures.

Global textures were recently proposed[1] as a new mechanism for cosmic structure formation. A phase transition in the early universe that breaks a continuous global symmetry ($G \rightarrow H$) would lead to configurations without a (macroscopic) coherence scale. A specific example are global textures[1,2], that arise when the homotopy group $\pi_3(G/H)$ is non-trivial. These structures collapse as soon as they come within the horizon, i.e. when they become causally connected. Their gravitational field would generate density inhomogeneities in an initially homogeneous matter distribution. In the simplest texture model[1] the symmetry $G = SU(2)$ is broken to $H = 1$ with $\pi_3(G/H \cong S^3) = Z$. The dynamics is governed by a Lagrangian

$$L = (\partial_\mu \Phi)(\partial^\mu \Phi)^\dagger - \frac{1}{4}\lambda(\Phi\Phi^\dagger - \eta_0^2)^2 \,, \tag{1}$$

where Φ for a spherically symmetric texture configuration is given by

$$\Phi = \exp\left(i\chi \, \frac{\vec{\sigma} \cdot \vec{r}}{r} \right) \cdot \eta \,, \tag{2}$$

with the radius vector $\vec{r}$ and the Pauli matrices σ_i. On scales $r \gtrsim m_\eta^{-1} = (\lambda\eta_0)^{-1}$, i.e. outside the core, the Higgs field $\eta(t, r)$ rapidly approaches its vacuum value and the Goldstone field $\chi(t, r)$ then parametrizes the motion on the vacuum manifold $|\eta| = \eta_0$. Therefore outside the core the potential term in (1) vanishes and the dynamics is solely determined by the scale-free gradient term. In absence of a scale, χ should be only a function of the self-similarity variable $x \equiv (t_* - t)/r \equiv \Delta t/r$, where t_* is the time at which the texture is collapsed. Space-time can then be viewed as the product space $R \times S^3$, i.e. the rays of constant x through the three-sphere of constant $\Delta t^2 + r^2$. The simplest Ansatz, a product of identity mappings of winding number ν from this sphere onto $S^3 \cong G/H$, is

$$\Phi = \left(\frac{\Delta t + i\,\vec{\sigma} \cdot \vec{r}}{\sqrt{\Delta t^2 + r^2}} \right)^{\nu} \cdot \eta \qquad (|\eta| = \eta_0) \,. \tag{3}$$

The equation of motion for this Φ becomes

$$2\,\nu\, x \, \frac{1 - x^2}{(1 + x^2)^2} + \Im\mathrm{m}\left(\frac{x + i}{\sqrt{1 + x^2}} \right)^{2\nu} = 0 \,. \tag{4}$$

Surprisingly, there is a solution for, and only for, $\nu = 2$ (besides the trivial mapping $\nu = 0$). This solution translates into

$$\chi = 2 \operatorname{arccot}(x) \qquad (-1 < x < +\infty) , \tag{5}$$

see fig. 1 [3] . Inversions and reflections of this are also solutions.

The solution describes the collapse and subsequent expansion of the texture configuration on scales larger than m_η^{-1}, i.e. well outside the core. This can be seen by looking at the energy momentum tensor

$$\begin{aligned} T_t{}^t &= 2\,\eta_0^2\, \frac{3\Delta t^2 + r^2}{(\Delta t^2 + r^2)^2} \\ T_r{}^r = T_\theta{}^\theta &= T_\phi{}^\phi = 2\,\eta_0^2\, \frac{\Delta t^2 - r^2}{(\Delta t^2 + r^2)^2} \, . \\ T_r{}^t &= -T_t{}^r = -4\,\eta_0^2\, \frac{\Delta t\, r}{(\Delta t^2 + r^2)^2} \end{aligned} \tag{6}$$

The solution is characterized by an isotropic pressure, which is negative in the time-like region and positive in the space-like region, with a maximum at $r = \sqrt{3}\Delta t$. The energy density is a monotonically decreasing function of r, but, for $t < t_0$, due to an inward energy flow it increases with time for $r < \sqrt{3}\Delta t$ and deceases outside. At $t = t_0$ the energy flow reverses direction and the solution then describes an expanding texture.

In the final state of the collapse, i.e. in the region $\Delta t^2 + r^2 < m_\eta^{-2}$, the Higgs field η also becomes dynamic. The fate of the texture knot is then determined by the coupled η-χ equations. A relaxation of η to η_0 in this final stage of the collapse would result in a release of (mainly) Goldstone bosons, which however would be insignificant compared to the energy contained in the coherent χ field. These dynamics determine the χ solution within the future light cone $x < -1$. Three possible extensions of (5) into the future light cone $x < -1$ are:

$$\begin{aligned} \chi_{(I)} &= 2 \operatorname{arccot}(x) && \text{(bounce)} \\ \chi_{(II)} &= 3\,\pi/\,2 && \text{(monopole)} \qquad (-\infty < x < -1) \, . \\ \chi_{(III)} &= 2 \operatorname{arccot}(1/x) && \text{(decay)} \, . \end{aligned} \tag{7}$$

Solution (I) is the elastic bounce of solution (5) , where the final texture configuration is the same as the initial. Solution (II) describes a static global (anti-) monopole[5] confined to $x < -1$, with an energy momentum tensor (outside the core)

$$T_t{}^t = T_r{}^r = \eta_0^2 / r^2 \,, \quad T_\theta{}^\theta = T_\phi{}^\phi = 0\,. \tag{8}$$

The solution (III) is obtained from (5) by inversion and describes the decay of the texture configuration into a $\chi = \pi$ vacuum. Solutions (II) and (III) have the same energy-momentum tensor (6) and action (modulo the small contribution from the texture core); thus their gravitational field is the same. All three solutions have the same mass M within the radius $r = -\Delta t$: $M = 4\pi\eta_0^2 r$.

The energy-momentum tensor (6) can now be used to calculate the gravitational field of the collapsing texture configuration. Since the texture collapse is scale-invariant (on scales larger than m_η^{-1}), the Einstein equations will contain only the dimensionless combination $\varepsilon \equiv 8\pi G\eta_0^2$ of the gravitational constant G and the symmetry breaking scale η_0. Since generally $\varepsilon \ll 1$, one can expand the metric around the vacuum solution, which here is Minkowski-space (neglecting the mass of the texture core and the expansion of the universe[6]).

The general spherical-symmetric Ansatz for the metric is

$$ds^2 = (1+\varepsilon\nu)\, dt^2 - (1+\varepsilon\lambda)\, dr^2 - (1+\varepsilon\omega)\, r^2\, (d\theta^2 + \sin^2\theta\, d\phi^2)\,, \tag{9}$$

where ν, λ and ω are functions of the self-similarity variable x[7] . In a coordinate system[8] with $\nu = 0$, the solution is[9] :

$$\begin{aligned} ds^2 &= dt^2 - \, dr^2 - \, r^2\, (1+\varepsilon\omega)\, (d\theta^2 + \sin^2\theta\, d\phi^2) \\ \omega &= \, -2 + \, 2\, x \arctan(1/x)\,. \end{aligned} \tag{10}$$

Since ω takes values between -2 and 0 this weak-field limit of the Einstein equations is an excellent approximation for *all* r and Δt. The gravitational effects of a collapsing texture are fully determined by this space-time dependent deficit solid angle $\omega(x)$. The solid angle of a sphere becomes locally $4\pi(1+\varepsilon\omega)$. The chosen coordinate system is distinguished by the fact that it preserves the meaning of the background time t and radius r, i.e. t and r remain the proper time and proper length. For the monopole as the final state, the energy-momentum tensor (8) gives a constant deficit angle[5] $\omega = -1$ in the region $x < -1$.

The immediate consequence of the metric (10) is that only matter with a non-zero angular momentum with respect to the texture origin experiences a force in the gravitational field. This can also be read off from the geodesic equations (for $\phi = const$):

$$\begin{aligned} \ddot{t} &= -\, r^2\, \dot{\theta}^2\, \varepsilon\, \omega_t\, /2 \\ \ddot{r} &= \dot{\theta}^2\left(r^2\,(1+\varepsilon\,\omega)\right)_{,r}/2 \\ \dot{\theta} &= L\,(1-\varepsilon\,\omega)\,/r^2 \qquad (L = const), \end{aligned} \tag{11}$$

where a dot represents the derivative with respect to the affine parameter along the geodesic and lower t and r *partial* derivatives.

With (11) it is now straightforward to derive the redshift of a photon passing through a texture configuration. To zeroth order in ε: $\dot{t} = E$ and $r = ((t-t_0)^2 + r_0^2)^{1/2}$. E is the frequency of the photon and t_0 and r_0 the time and radius of its closest approch to the texture origin, see fig. 1. To first order one then finds

$$\dot{t} = \; E\left(1 - \frac{1}{2}\varepsilon r_0^2 \int \, dt\,\omega_t/r^2\right). \tag{12}$$

Since the result should be of first order in ε, the integral can be evaluated along the zeroth order photon path. Integrating by parts one finds for the fractional redshift (for $r_0 > 0$)

$$\begin{aligned} \delta E/E = \frac{\dot{t}-E}{E} = -\varepsilon\Bigg[& \frac{t_* - t_0}{\sqrt{(t_*-t_0)^2 + 2r_0^2}}\,\mathrm{arccot}\left(\frac{t_* + t_0 - 2t}{\sqrt{(t_*-t_0)^2+2r_0^2}}\right) \\ & + \frac{t_0 - t}{\sqrt{(t-t_0)^2 + r_0^2}}\,\arctan\left(\frac{\sqrt{(t-t_0)^2+r_0^2}}{t_* - t}\right) - \frac{\pi}{4}\Bigg]\,. \end{aligned} \tag{13}$$

where for the solution (II) the r.h.s. has to be multiplied by the step function $\theta(t_* - t_0)$.

Up to here the gravitational field of textures was calculated over a flat background, which is a good approximation on scales $(\Delta t^2 + r_0^2)^{1/2} \lesssim H^{-1} = t/2$, where H^{-1} is the cosmic expansion rate and t the conformal cosmic time for a (flat) dust universe. However, on scales $r \gtrsim t/2$ the texture configuration would be incoherent, and on scales $\Delta t \gtrsim t/2$ static. For $-\Delta t \gtrsim t/2$ the effects from other textures would have to be included. It is therefore reasonable to limit the analysis of a single texture configuration to the region:

$$(\Delta t^2 + r^2) \lesssim t/2\,, \tag{14}$$

We denote the times when a photon passes into and out of this region by t_{in} and t_{out}. The fractional redshift distortion of microwave background (MBR) photons due to a texture is then $\delta E/E(t = t_{out}) - \delta E/E(t = t_{in})$. Only photons with impact paramters $r_0 \lesssim t_*/2$ and $t_*/2 \lesssim t_0 \lesssim 3t_*/2$ will then experience the gravitational field of the collapsing texture. One finds that the fractional redshift distortion is always positive (blueshift) for $t_0 < t_*$ and negative (redshift) for $t_0 > t_*$. For the solution (III) there are only blueshifts induced by (5) in the region $t_0 < t_*$. The maximum values of the fractional blue- and redshifts of $\pm\varepsilon\pi$ occur for $r_0 \to 0$ (independent of t_{in} and t_{out}).

Since a texture collapse is expected to occur every few horizon scales[2,1] , one would expect distortions of the isotropy of the MBR of the order $O(\varepsilon)$ on all angle scales above the horizon size at recombination $\theta_{rec} \approx 1°$. The measured isotropy

$$\delta T/T\Big|_{\theta \gtrsim 1°} \lesssim 6 \times 10^{-5}\ . \tag{15}$$

limits $\varepsilon \lesssim 2 \times 10^{-5}$ or the symmetry breaking scale to $\eta_0 \lesssim 1 \times 10^{16} GeV$.[10]

The geodesic equations (11) show that there is no force on matter moving radially with respect to the texture origin. In particular, there is no force on particles at rest. However, since the texture metric is time-dependent it changes the volume of three-space and thereby also modifies the matter density. For an initially homogeneous and static mass density ρ_0 the induced fluctuations are

$$\delta\rho/\rho = \frac{\rho(r,t) - \rho_0}{\rho_0} = -\varepsilon\,\omega(r,t). \tag{16}$$

The considered textures thus generate overdensities by contracting the three-space volume via the negative deficit angle ω given in (10) . Since texture effects are limited by (14) to the three-space region $r \lesssim t_*/\sqrt{3}$, this could lead to horizon size overdense shells in the matter distribution.

These density fluctuations would be only temporary, if there were no gravitational forces. For small rotational velocities $v_\theta = L/r_0$ and $r \approx r_0$ the acceleration in addition to the zeroth order centrifugal force is

$$\Delta \ddot{r} = \varepsilon \, \frac{v_\theta^2}{r_0} \left(2 + 3\,x \, \arctan(1/x) - x^2/(1+x^2)\right) \geq 0 \ \text{ with } \ x = (t_* - t_0)/r_0 \, . \quad (17)$$

The considered textures therefore give rise to a repulsive force. This is mainly due to an increase of the rotational velocity of the surrounding matter. This effect could in fact generate inhomogeneities in an initially homogeneous and static matter distribution, when textures *move* with respect to the matter rest system. The combined effect of the density enhancement by volume contraction and the repulsive force due to an increase in the rotational velocity might lead to outward moving overdense shells of matter[11].

In the process of writing up the above results I became aware of the preprint in Ref. 4. The limit $t \to +\infty$ of (13) agrees with the frequency shift formula there. However, the results for the gravitational forces on matter differ.

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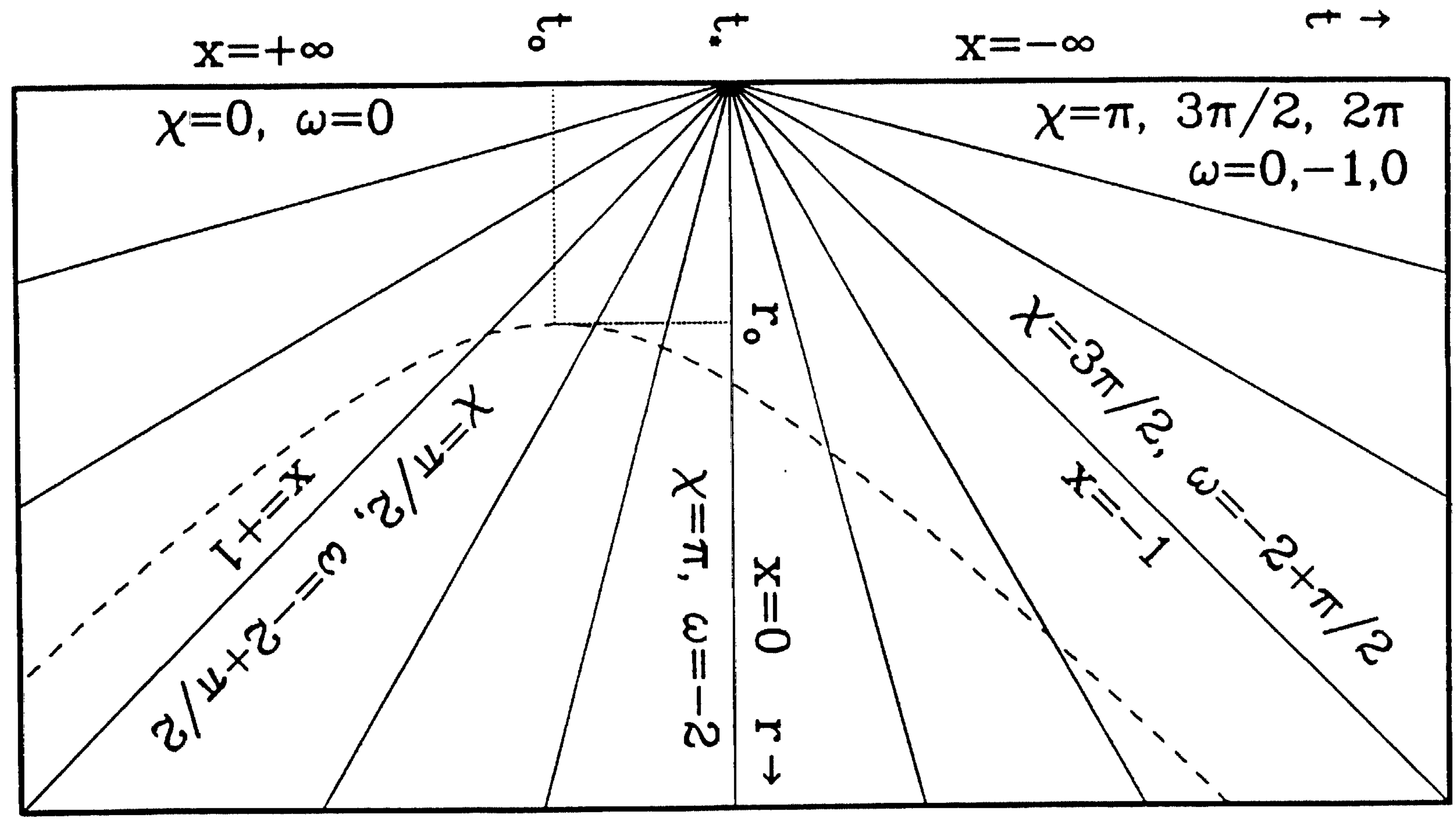
3. This solution, which is the same as in Ref. 4, was obtained after David Spergel told me that he had found a simple exact analytic solution (private communication, Dec. 1989).
4. D. Spergel and N. Turok, Princeton Univ. preprint, PUPT-90-1167, (1990)
5. M. Barriola and A. Vilenkin, *Phys. Rev. Lett.* **63**, 341, (1989)
6. The mass contribution from the texture knot, $\sim \eta_0/\lambda^2$, can be neglected. Also η converges rapidly to η_0 on scales $\Delta t, r$ larger than m_η^{-1}: $\eta \approx \eta_0\ (1 + 4(r^2 - 3\Delta t^2)/(m_\eta^2\ (\Delta t^2 + r^2)^2))$ for solutions (I) and (III) and $\eta \approx \eta_0\ (1 - 1/(m_\eta^2 r^2))$ for solution (II). On scales smaller than the horizon also the cosmic expansion and curvature are unimportant.
7. Here the assumption of self-similarity means no loss of generality.
8. Since the pressure is isotropic, only two coefficient functions are independent.
9. The details will be reported elsewhere.
10. A detailed analysis of the induced fluctuation spectrum on the MBR will be made in a future publication, in preparation.
11. This effect will be worked out in a seperate paper, in preparation.

FIGURE CAPTIONS

1. Space-time diagram for the texture solutions eq. (5) and (7). By self-similarity χ is constant along the lines of constant $x = (t_* - t)/r$. The dashed curve represents the path of a photon.


t →
x=+∞
x=−∞
t_o
t_*
χ=0, ω=0
χ=π, 3π/2, 2π
ω=0,−1,0
r_o
χ=3π/2, ω=−2+π/2
x=−1
χ=π, ω=−2
x=0
r →
χ=π/2, ω=−2+π/2
x=+1